$S_{|\{e,s,t,i,n,a\}|}$

by CALEB EMMONS

Definition 1: To achieve the poetry form
Celebrated for its symmetries
And known far and wide as the sestina
The concluding words of the first six
Lines must comprise a distinguished group,
Ending subsequent lines in prescribed permutations.

Definition 2: What precisely is meant by permutations?
The set of rearrangements of n objects form
S_n, the so-called symmetric group
Which captures all finite symmetries.
(Previously we chose $n = 6$
When we defined the sestina.)

Question: If we distill from a sestina
The sestets' corresponding permutations
(Of which there are six)
And out of these form
A subgroup of symmetries
Have we recovered the whole group?

Theorem: Working in the symmetric group
If we reduce a sestina
To its bare symmetries
And gather those permutations
The subgroup they form
Is cyclic of order six.

Proof: Let τ be the cycle (1 2 4 5 3 6).
By mapping integer k to group
Element τ^{k-1} it's easy to check that we form
A bijection from the sestets of the sestina
To their corresponding permutations.
(The work can be reduced by noticing symmetries.)

Corollary: Because of these symmetries
If you've written only two sestets of six,
With their rigidly fixed permutations,
Nonetheless, you may shift this group
To elsewhere in your sestina
And retain their form.

Erratum: In all our discussion of permutations and poetic symmetries
We neglected to mention that the form has, in addition to the six
Sestets, another group of lines: a final tercet to complete the sestina.